abacus
THE QUANT CLUB OF IIMA

# ABACUS Tesseract 2013 – Set #10

## Question #1:

*There were two men having a meal. The first man brought 5 loaves of bread, and the second brought 3. A third man, Ali, came and joined them. They together ate the whole 8 loaves. As he left Ali gave the men 8 coins as a thank you. The first man said that he would take 5 of the coins and give his partner 3, but the second man refused and asked for the half of the sum (i.e. 4 coins) as an equal division. The first one refused.*

*They went to Ali and asked for the fair solution. Ali told the second man, "I think it is better for you to accept your partner's offer." But the man refused and asked for justice. So Ali said, "then I say that who offered 5 loaves takes 7 coins, and who offered 3 loaves takes 1 coin." Can you explain why this was actually fair?*

## Question #2:

*A woman wants to buy a painting at an auction where you bid grams of gold instead of money. She owns a gold chain made of 23 interlocking loops, each weighing 1 gram. She wants to go to a jeweller before the auction to cut the minimum number of loops that would allow her to pay any sum from 1 to 23. For example, she could pay a 13 gram price with a 12 link chain and a single link. After much thought, she figures out a way to do it by cutting just 2 of the loops in the chain. How many loops are in the pieces of chains that she has after the 2 cuts?*

*When she cuts a loop in the middle it gets separated from the portion of chain connected on the left side and on the right side. Hence one cut results in three pieces of chain.*

## Question #3:

*A bad king has a cellar of 1000 bottles of delightful and very expensive wine. A neighboring queen plots to kill the bad king and sends a servant to poison the wine. (un)Fortunately the bad king's guards catch the servant after he has only poisoned one bottle. Alas, the guards don't know which bottle but know that the poison is so strong that even if diluted 1,000,000 times it would still kill the king. Furthermore, it takes one month to have an effect. The bad king decides he will get some of the prisoners in his vast dungeons to drink the wine. Being a clever bad king he knows he needs to murder no more than 10 prisoners - believing he can fob off such a low death rate - and will still be able to drink the rest of the wine at his anniversary party in 5 weeks' time.*

*Additional question: To increase your chances of living, which prisoner would you want to be?*

## SOLUTIONS

**Solution 1:**

The question is why give them 1 and 7 coins when they have brought 3 and 5 loaves respectively. The men may actually have brought 5 and 3 loaves but they have also eaten something too.

It is reasonable to think that the 3 men shared the loaves equally, eating 2 ⅔ loaves each, so the actual contributions of the two persons to the 3rd person are:

Person #1: 5 - 2 ⅔ = 2 ⅓

Person #2: 3 - 2 ⅔ = ⅓

Person #1 gave 2 ⅓ loaves, or looking at it in thirds they gave 7 thirds as opposed to person #2 who gave just 1 third. Hence, it is fair to divide the money in the ratio 7:1

**Solution 2:**

Cut the 4th and 11th loops from the gold chain which will give us 4 chains with following number of rings: 3, 1, 6, 1, 12. Using these 4 chains the woman can weigh from 1 to 23 grams.

1 + 1 = 2

3 + 1 = 4

3 + 1 + 1 = 5 and so on…

**Solution 3:**

Label the wine bottles from 1 to 1000. Each of the bottle number can be represented by a binary 10 bit number (since the maximum that can be represented by 10 bits is 2 power 10 = 1024).

Mark each of prisoners from 1 to 10.

Do the following for each of the bottles 1 to 1000:

Step 1: Take note of the bottle number

Step 2: In binary format check which bits are set to 1 and make those particular numbered persons drink that bottle

Example: 6 = 00 0000 0110

So person numbered 2 and 3 will drink the bottle.

After the above 2 steps are performed line the prisoners up in their bit order and read each living prisoner as a 0 bit and each dead prisoner as a 1 bit. The number that you get is the bottle of wine that was poisoned.

Example: Say number 4 and number 6 prisoners are dead which is equivalent to the following binary: 0000101000 which is 40 in decimal. Bottle numbered 40 is the poisoned bottle.

Additional question's answer

If there were 1023 bottles, it wouldn't matter since everyone would have to take 512 sips. But there are 23 bottles less, so the people whose bits would have been on from 1001 to 1023 won't have to take a sip. 1001 is [11111 01001] in binary and 1023 is [11111 11111]. The most five significant bits are the most interesting because they would always be on from 1001 to 1023, so all those people are missing out on 23 bottles of wine that they otherwise would have had to drink. So in order to increase your chance of living, you'd probably want to be prisoner 6 to 10. (But depending on how the king determines who is least significant and who is most significant you could get shafted.)