

ABACUS Tesseract 2013 – Set #12

Question #1:

A Tower has 10 floors. Each needs to be painted with either pink or yellow color. But no two adjacent floors are to be colored pink. In how many ways can the floors be painted?

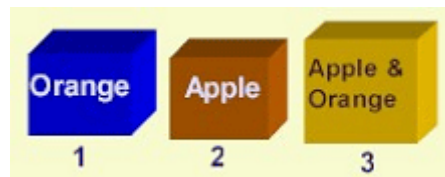
Question #2:

In a 4000 meter race around a circular stadium having a circumference of 1000 meters, the fastest runner and the slowest runner reach the same point at the end of the 5th minute, for the first time after the start of the race. All the runners have the same starting point and each runner maintains a uniform speed throughout the race. If the fastest runner runs at twice the speed of the slowest runner, what is the time taken by the fastest runner to finish the race?

1. 20 min 2. 15 min 3. 10 min 4. 5 min

Question #3:

Boxes of Apples and Oranges



All 3 boxes with labels are COMPLETELY mislabeled. You will be allowed to take only one piece of fruit from one of the boxes to examine it. Which box would you choose? How do you correctly re-label all three boxes?

SOLUTIONS

Solution 1:

Let A_{n+1} be the number of ways to paint $n+1$ floors. A_{n+1} will include the following 2 cases:

- **When 1st floor was painted pink.** Here the next floor has to be yellow. So the remaining $n-1$ floors can be painted in A_{n-1} ways
- **When 1st floor was painted yellow.** Here the remaining n floors can be painted in A_n ways.

Hence we get the following recursion: $A_{n+1} = A_n + A_{n-1}$ (familiar Fibonacci expression)

With $A_1 = 2, A_2 = 3$... (For these small nos. we can calculate answer manually)

So, the pattern can be obtained as : 2, 3, 5, 8, 13, 21, 34, 55, 89, **144**.

Solution 2:

The ratio of the speeds of the fastest and the slowest runners is 2 : 1. Hence they should meet at only one point on the circumference i.e. the starting point (As the difference in the ratio in reduced form is 1). For the two of them to meet for the first time, the faster should have completed one complete round over the slower one. Since the two of them meet for the first time after 5 min, the faster one should have completed 2 rounds (i.e. 2000 m) and the slower one should have completed 1 round. (i.e. 1000 m) in this time. Thus, the faster one would complete the race (i.e. 4000 m) in 10 min.

Solution 3:

This puzzle is very simple if you list out all the cases. Remember that ALL the boxes are wrongly labeled.

1. Orange box (box 1) can have apples or "apples and oranges".
2. Apple box (box 2) can have oranges or "apples and oranges".
3. Mixed box (box 3) can have either apples or oranges but not both.

Let us try the orange box (box 1) first. If you are lucky, you pick up an orange, so you can be sure you also have an apple in the box (mixed). In this case the problem is solved. Unfortunately, you have only a 25% chance of getting an orange. If you pick up an apple instead, you can not tell if there may also be oranges in the box.

If you select the Apple box (box 2), it will be the same as selecting the orange box.

The only choice is to select from the box labeled "mixed" (box 3).

We know it is mislabeled so that it CANNOT contain mixed fruit. If you pick up an orange, then you know that the box labeled "apples" must contain the mixed fruits. (If it had just apples in it, it would not be mislabeled!) The orange box can have apples only. If you pick up an apple, the apple box can have oranges only. The orange box will then be the mixed fruit.