

ABACUS Tesseract 2013 – Set #13

Question #1:

5 pirates of different ages have a treasure of 100 gold coins. They decide to split the coins using this scheme:

The oldest pirate proposes how to share the coins, and all pirates remaining will vote for or against it. If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain.

Assuming that all 5 pirates are intelligent, rational, greedy, and do not wish to die, (and are rather good at math for pirates) what will happen?

Question #2:

A pirate ship has a crew of 14 members including its captain and a treasure box. No two locks can be unlocked by the same key and all the locks need to be unlocked to open the box. Determine the minimum number of locks and copies of keys that will be required to be distributed between the members of the crew such that, any six or fewer members of the crew cannot open the box but any combination of seven can.

Question #3:

*A fachha (Mr. Dude) writes a **JOOS** to a fachhi (Miss Dudette). He concludes his JOOS with an unusual request:*

“Let’s meet at CT @9. I will make a statement. The statement is either true or false. If it is true, promise that you will give me one of your photographs. If it is not true, promise that you will not give me your photograph.”

This fachhi ain’t no spoil sport so she agrees to go to CT and hear out his statement. She also promises to play by the rules and carries a photograph along with her.

*Now what should our fachha say to this fachhi, so that to keep her promise, the **fachhi has to KISS** him.*

SOLUTIONS

Solution 1:

Answer - The eldest pirate will propose a 97 : 0 : 1 : 0 : 2 split.

Working backwards, splits in terms of younger to older:

2 Pirates: Pirate Two splits the coins 100 : 0 (giving all to the other pirate). Otherwise, and perhaps even then, Pirate One (the youngest) would vote against him and over he goes!

3 Pirates: Pirate Three splits the coins 0 : 1 : 99. Pirate One (the youngest) is going to vote against him no matter what (see above), but this way, Pirate Two will vote for him, to get at least one gold out of it.

4 Pirates: Pirate Four splits the coins 1 : 2 : 0 : 97. This way, Pirate One will vote for him, and so will Pirate Two - they're getting more than they would under 3 pirates.

5 Pirates: Pirate five splits the coins 2 : 0 : 1 : 0 : 97. This way, Pirate One will vote for him, and so will Pirate Three - they're both getting better than they would under 4.

Solution 2:

Let me start small and explain. Suppose there were 6 pirates and you want any three to open the lock (s) but not any two. How many locks do you need and how many keys do you need to distribute among the pirates?

First, how many groups of two pirates from 6 pirates A, B, C, D, E and F can you make? It will be ${}^6C_2 = 15$. The groups are AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, and EF. We want that none of these groups should be able to open a lock since no groups of two should open a lock. Therefore, for minimum number of locks, we keep one lock for each group that the group is not able to open. i.e. let's say AB is not able to open K_1 , AC is not able to open K_2 , ...EF is not able to open K_{15} . So for K_1 , we will give the keys to every pirate except AB, for K_2 we will give keys to every pirate except AC... for K_{15} we will give the keys to every pirate except EF. This way, every group of two will be able to open 14 not locks but not the 15th one. But if it combines with other third person, it will be able to open all the locks! Therefore, the number of locks needed is 15 and every pirate will not have those many as equal to the number of groups of two he figures in. For example AB will not have keys for K_1 , AC will not have keys for K_2 , AD will not have for K_3 ..and so on for groups containing A. therefore, A will not have keys for K_1, K_2, K_3, K_4, K_5 but will have keys for rest of the groups. So A will have 10 keys. Similarly, for your puzzle, you will need to form groups of 6 in ${}^{14}C_6$ ways. These are the number of locks you will need, i.e. one for each group. Therefore, you will need 3003 locks. And each pirate will have all the keys except the number of keys equal to the number of groups of 6 he does **NOT** figure in, i.e. ${}^{13}C_6 = 1716$.

Solution 3:

“Neither will you give me your photograph, nor will you kiss me”

Assume the above statement is true, then she has to give him her photo. But the statement itself says she will not give her photo. So we have a contradiction. So, the above statement cannot be true.

Therefore the above statement cannot be true.

Therefore the statement is false. (Given that the statement is either true or false)

So for above statement to be false she will have to give either her photo or a kiss. But she has promised she will not give her photo in case the statement is false. So she will have to give him a kiss.