ABACUS Tesseract 2013 – Set #14

Question #1:

Ajay and Bikram are playing a game. They start with the number 1 and take turns to multiply it by integers between 2 and 9 (both included). The same multiplier can be used again in subsequent turns. The first one to take the product above 1000 wins. If Ajay has the first turn, what are the multipliers he can use in his first turn to ensure victory?

Question #2:

A person dies, and arrives at the gate to heaven. There are three doors. One of them leads to heaven. Another one leads to a 1-day stay at hell, and then back to the gate, and the other leads to a 2-day stay at hell, and then back to the gate. Every time the person is back at the gate, the three doors are reshuffled. How long will it take the person to reach heaven?

Question #3:

A king held swayamwar for his daughter and called upon 3 most intelligent men A, B and C from his kingdom. He showed them 5 caps – 3 White and 2 Black. He asked them that he will give 1 cap to each and will hide 2 caps. Participant had to recognize the color of cap he is wearing on following Conditions:

1. No one can look at his cap's color
2. Participants can see color of cap wore by other two guys.
3. Whoever answers first will marry king's daughter
4. If the first answer given by any participant is incorrect than that will lead to severe punishment.

After quite some time, A answered the question and that was correct.
A saw that B and C were wearing White color caps. What was A's answer?

Question #4:

The warden meets with 23 new prisoners when they arrive. He tells them, "You may meet today and plan a strategy. But after today, you will be in isolated cells and will have no communication with one another.

"In the prison is a switch room, which contains two light switcheslabeled 1 and 2, each of which can be in either up or the down position. I am not telling you their present positions. The switches are not connected to anything.

"After today, from time to time whenever I feel so inclined, I will select one prisoner at random and escort him to the switch room. This prisoner will select one of the two switches and reverse its position. He must flip one switch when he visits the switch room, and may only flip one of the switches. Then he'll be led back to his cell.

"No one else will be allowed to alter the switches until I lead the next prisoner into the switch room. I'm going to choose prisoners at random. I may choose the same guy three times in a row, or I may jump around and come back. I will not touch the switches, if I wanted you dead you would already be dead.

"Given enough time, everyone will eventually visit the switch room the same number of times as everyone else. At any time, anyone may declare to me, 'We have all visited the switch room.'

"If it is true, then you will all be set free. If it is false, and somebody has not yet visited the switch room, you will all die horribly. You will be carefully monitored, and any attempt to break any of these rules will result in instant death to all of you"

What is the strategy they come up with so that they can be free?

SOLUTIONS

Solution 1:

Answer - 4,5,6

Note that 9*111 = 999 and 9*112 > 1000. So if Ajay gets a number greater than 112 in his turn, he will definitely win. So he has to ensure he plays the game such that Bikram is forced to give him a number >= 112. Now backtracking 112/2 = 56.
If A gives B a no. between 56 and 111, B will not be able to win in his turn, but in the very next turn A will definitely win. So the aim is to give B a no. between 56 and 111.
If A gives B any of the nos. 4,5,6 then B will not be able to take the product above 56 in his turn, but in the very next turn A will definitely be able to do so.

Solution 2:

1/3 of the time, the door to heaven will be chosen, so 1/3 of the time it will take zero days. 1/3 of the time, the 1-day door is chosen; of those, the right door will be chosen the next day, so 1/9 trips take 1 day.
Similarly, 1/9 will take two days (choosing the 2-day door, then the right door).
After that, the cases split again, and again, and again. I can't seem to make a nice infinite sum this way, so let's try again. Suppose the average days spent is E. 1/3 of the cases are done in zero
days as before.
1/3 of the cases are 1 day plus E. 1/3 are 2 + E.
So:
E = 1/3 * 0 + 1/3 * (1 + E) + 1/3 * (2 + E)
= 0 + 1/3 + E/3 + 2/3 + E/3
= 1 + 2E/3
Therefore,
E/3 = 1
E = 3
On average, it takes three days to get to heaven

Solution 3:

A is wearing a white hat.

Assume that A is wearing a black hat.

B's thought process : I can see a black hat (A) and a white hat (C). So the colour of my hat could be either white or black. But if I am wearing a black hat, then C must be looking at 2 black hats - so C should be able to quickly conclude that he is wearing a white hat (since there are only 2 black hats)...but C is silent...so I am not wearing a black hat ... so i am wearing a white hat.

Similarly, we can write C's thought process. He too will be able to conclude that the colour of his hat is white (again B's silencehelps).

But clearly this is not the case. We are given that neither B nor C speaks up. So the assumption that A is wearing a Black hat leads to a contradiction.

So, A is wearing a white hat. So all 3 are wearing white hats. The game is even and fair - all 3 have equal chance of winning the bride.

A is clearly the smartest of the lot. He was intrigued by B and C's silence. He tried to figure out what B and C's thought process would be if he was wearing a black hat and realised that if this were the case then either of B or C would have easily replied by now - this delay was unwarranted. So he concluded that he was wearing a white hat. Note that similar reasoning could have been used by B and C as well and they too could have questioned the silence of their peers and answered correctly that the colour of their hat is white. That is why it is a fair test - appropriate to choose the groom. A won because he was smarter than the other 2 and hence was able to figure out sooner than his peers.

So the "silence" of the peers was a vital piece of information. Apart from the colours of the hats 'that one could see', one needed to factor in 'what was not said' to answer correctly

Solution 4:

The team nominates a leader. The group agrees upon the following rules:

The leader is the only person who will announce that everyone has visited the switch room. All the prisoners (except for the leader) will flip the first switch up at their very first opportunity, and again on the second opportunity. If the first switch is already up, or they have already flipped the first switch up two times, they will then flip the second switch. Only the leader may flip the first switch down, if the first switch is already down, then the leader will flip the second switch. The leader remembers how many times he has flipped the first switch down. Once the leader has flipped the first switch down 44 times, he announces that all have visited the room.

It does not matter how many times a prisoner has visited the room, in which order the prisoners were sent or even if the first switch was initially up. Once the leader has flipped the switch down 44 times then the leader knows everyone has visited the room. If the switch was initially down, then all 22 prisoners will flip the switch up twice. If the switch was initially up, then there will be one prisoner who only flips the switch up once and the rest will flip it up twice.

The prisoners can not be certain that all have visited the room after the leader flips the switch down 23 times, as the first 12 prisoners plus the leader might be taken to the room 24 times before anyone else is allowed into the room. Because the initial state of the switch might be up, the prisoners must flip the first switch up twice. If they decide to flip it up only once, the leader will not know if he should count to 22 or 23.

In the example of three prisoners, the leader must flip the first switch down three times to be sure all prisoners have visited the room, twice for the two other prisoners and once more in case the switch was initially up.