abacus
THE QUANT CLUB OF IIMA

ABACUS Tesseract 2013 – Set #1

Question #1:

You are given a fair cubical dice with 6 sides with 1 to 6 printed on each side. Suppose you are given three attempts to roll the dice. You may stop rolling the dice after the first attempt or after the first two attempts or after all the 3 attempts. Whenever you stop, the number on the side facing up is your score. If you wish to get the maximum score, what should be your strategy?

Question #2:

Imagine the GS markets interview that consists of six interviewees sitting side by side along one side of a table, with you, the candidate, at one end. When the interviewer says "go!" everyone at the table begins a conversation with the person either to the left or to the right. Assuming that, on the command, people (except the ones at the ends, who have no choice) turn to either their right or their left at random, and assuming that those who find a partner at once in this way stick with that partner, and assuming that those who don't find a partner at once will behave rationally, doing their best to pair off with someone to their left or right, what is the probability that you will find yourself with no one to talk to?

Question #3:

A blind-folded man is handed a deck of 52 cards and told that exactly 10 of these cards are facing up. How can he divide the cards into two piles (possibly of different sizes) with each pile having the same number of cards facing up?

SOLUTIONS

Solution 1:

The expected value of the number that would show up on the dice in any attempt would be 3.5. Now let us work backwards. Suppose your score is x after 2 attempts, i.e. you have rolled twice and the dice showed x on the 2nd attempt. Clearly, if x > 3.5, you would not go for the 3rd attempt, i.e., if you get 1, 2, or 3 in the 2nd attempt, you will go for the 3rd attempt, otherwise not. Now suppose your score is w in the 1st attempt. The expected score if you do not go for the 2nd attempt is 3.5; if you go for the 2nd attempt, then the expected score would be calculated as follows:

(1/2) * 5 + (1/2) * 3.5 = 4.25

This is because if you get 1, 2, or 3 in the 2nd attempt (with probability of 1/2), you will go for the 3rd attempt and your expected score would be 3.5. If you get 4, 5, or 6 in the 2nd attempt (with probability of 1/2), you will stop after the 2nd attempt and your expected score would be 5. Therefore, if w > 4.25, you would not go for the 2nd attempt, i.e., if you get 1, 2, 3, or 4 in the 1st attempt, you will go for the 2nd attempt, otherwise not.

Solution 2:

2 important things to be kept in mind: initially, the middle 5 interviewees behave randomly, and later they behave rationally. Initially, after the word 'go', there are 2^5 = 32 possibilites some of which are as follows:

RRRRRRL

RRLRRRL

RRLLLRL

RRRRRLL

Note that candidates at extreme left and right always turn right and left respectively. So the sequence of the middle 5 are as follows:

RRRRR

RLRRR

RLLLR

RRRRL

and so on.

16 of these 32 possibilities begin with L. In these cases, you strike up a conversation with B. Of the remaining 16, 8 begin with RL. In these cases, B strikes up a conversation with C. You are left out. The remaining 8 are as follows:

RRRRR

RRRRL

RRRLR

RRRLL

RRLRR

RRLRL

RRLLR

RRLLL

Of these 8, the 2 that begin with RRRL, D & E start conversation. B has a choice to either talk to A or C. We can assume that it is equally likely that B talks to A or C. So in 1 of these 2 cases, A would be left out.

In all the other cases, B would act rationally and turn to you.

Hence, probability that you would find yourself with no one to talk to = **(8 + 1) / 32 = 9/32**

Solution 3:

All he has to do is divide the deck in **2 piles of 10 and 42 cards** respectively and flip all the cards in the smaller pile.