ABACUS Tesseract 2013 – Set #2

Question #1:

There are 10 sets of 10 coins. You know how much the coins should weigh. You know all the coins in one set of ten are exactly a hundredth of an ounce off, making the entire set of ten coins a tenth of an ounce off. You also know that all the other coins weight the correct amount. You are allowed to use an extremely accurate digital weighing machine only once. How do you determine which set of 10 coins is faulty?

Question #2:

You are given 81 coins and are told that one of the coins is lighter than the rest. You are also given a pair of balance scales. What strategy will you choose to find out the odd coin in which the balance is used for the minimum number of times?

Question #3:

A and B are playing a game. 100 numbers are listed one after the other horizontally on a piece of paper. A plays first and is given the choice of picking one of the numbers from either end. On his turn B is given the choice of picking one of the numbers from either end of the remaining numbers.The process continues till all the numbers are exhausted. A and B independently sum up the numbers they have chosen and the person with the larger sum is declared as the winner. What strategy should A use (given that he plays first) so that he never loses?

SOLUTIONS

Solution 1:

Assume each coin weighs "x" ounces and a coin in the faulty set weighs "x+0.01" ounces.
Take one coin from 1st set, 2 coins from 2nd set and so on.
Weigh these 55 coins, if you obtain "55x+0.01" ounces clearly the1st set is faulty, "55x+0.02" ounces would mean 2nd set is faulty and so on.

Solution 2:

This problem can be generalized. 3^n coins can be handled in n weighings by the following method:
a) Place 3^(n-1) coins on each side of the balance and determine which group of 3^(n-1) coins contains the light coin.
b) Having determined which group of 3^(n-1) contains the light coin, subdivide it into three equal 3^(n-2) groups of each and place one of these on each side and thereby determine which group of 3^(n-2) contains the light coin.
c) By continuing this process the size of the group can be reduced by two-thirds at each successive weighing. Therefore n weighings will be able to handle 3^n coins.
For 81 coins, we need 4 weighings.

Solution 3:

Add all numbers in even positions and add all in odd positions. Whichever is higher pick from that end. (If odd positions give a higher sum, pick the number in 1st position. Opponent will pick from either the 2nd or 100th positions, You can pick from 3rd or 99th position and so on. You will end up winning. Similarly if even positions give a higher sum, pick the number in 100th position)