ABACUS – Summers Prep 2013 – Set#3

Question #1:

You and I are to play a competitive game. We shall take it in turns to call out integers. The first person to call out fifty wins. The player who starts must call out an integer between 1 and 10, inclusive. A new number called out must exceed the most recent number by at least one and by no more than 10. What is your winning strategy if you are going first?

Question #2:

You are standing at the foot of a 200 storey building with two absolutely identical eggs. You can throw the egg from any floor and see if it was broken or not. If not, you can reuse it again. Identify the minimum possible throws required in the worst case to find out the "breaking floor" (Lowest floor from which the eggs are broken if thrown)

Question #3:

You are to open a safe without knowing the combination. Beginning with the dial set at zero, the dial must be turned counter-clockwise to the first combination number, (then clockwise back to zero), and clockwise to the second combination number, (then counter-clockwise back to zero), and counter-clockwise again to the third and final number, where upon the door shall immediately spring open. There are 40 numbers on the dial, including the zero.
Without knowing the combination numbers, what is the maximum number of trials required to open the safe (one trial equals one attempt to dial a full three-number combination)?

Question #4:

An aircraft has exactly 100 seats for passengers. 100 passengers are waiting in a line outside this aircraft, their boarding cards in their hands. Now the first passenger gets onto the plane. He drops his boarding card on purpose and chooses a seat at random. When the second passenger comes in, if he sees that his seat is occupied, he chooses another seat at random. If his seat is not occupied, he takes his own seat. Like this, one by one, 99 passengers board the plane. If one finds one's seat occupied, he chooses a seat at random. Finally, the 100th passenger enters the plane. There is only one seat left in the plane - he has to take it. What is the probability that he gets the seat assigned to him on his boarding card?

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Solution #1:

This can be solved by thinking backwards. As my opponent, if you call out 39, I cannot reach fifty and you will win. Similarly, to be able to call 39, you should call 28, so that I am unable to call 39. To call 28, you should be able to call 17 and to call 17 you should start with 6. So 6,17,28,39,50 are the winning numbers.

Solution #2:

Minimum no. of throws required – 20. One way to do this is as below
- Throw the egg from 20th floor
- If it breaks
· Then start throwing the other egg beginning from 1st floor
· No. of throws required in worst case – 20 [1 (20th floor trial) + 19]
- If it doesn't break, throw the egg from 39th floor (20+19)
- If it breaks
· Then start throwing the other egg beginning from 21st floor
· No. of throws required in worst case – 20 [2 (20th & 39th floor) + 18 (floor 21 to 38)]
- If it doesn't break, throw the egg from 57th floor (20+19+18)
Repeat this process till you cover all the 200 floors
In general if there are "x" floors, the minimum number of throws required will be equal to n
where n is the lowest value satisfying $n*(n+1)/2 >= x$

Solution #3:

The key word here is 'immediately.'
The implication of this is that you do not have to try 40 times at the last number for each combination of the first number two numbers.
With this in mind you see that after any combination of the first two numbers you can, instead of trying all of the 40 possibilities for the last number, just turn the dial all the way to the end for the last number; in doing this you will necessarily pass the correct number where upon 'the door shall immediately spring open.'
40 x 40 = 1600

Solution #4:

Solution:

It is clear that as soon as the assigned seat of the first passenger is occupied, the remaining of the passengers would occupy their assigned seats without any trouble.

Therefore, if the assigned seat of the first passenger is occupied before the assigned seat of the last passenger is occupied, the last passenger would get the seat assigned to him.
The 1st passenger enters. There is equal probability that he occupies his assigned seat or he occupies the seat assigned to the last passenger. Suppose he occupies seat # i.

Next (i-2) passengers will occupy seats assigned to them. When the ith passenger enters, again there is equal probability that he occupies the seat assigned to the first passenger or he occupies the seat assigned to the last passenger.
Therefore at any point when a passenger sees his assigned seat to be occupied already, there would be equal probability that he occupies the seat assigned to the first passenger or he occupies the seat assigned to the last passenger.
Therefore, there is equal probability that the assigned seat of the first passenger is occupied before the assigned seat of the last passenger is occupied.
Thus, probability that the last passenger gets the seat assigned to him on his boarding card is 1/2.