

ABACUS – Summers Prep 2013 – Set#4

Question #1:

You hire a worker to work for you for seven days. In return for his work, you will pay him $\frac{1}{7}$ th of a bar of gold per day. The worker requires a daily payment of $\frac{1}{7}$ th of the bar of gold. What and where are the fewest number of cuts to the bar of gold that will allow you to pay him $\frac{1}{7}$ th each day? On successive days, he may use what you paid him previously to make change.

Question #2:

Three out of six lookalike balls are heavy. The other three are light. How many weighings on a beam balance are necessary to identify the heavy balls?

Question #3:

Three Palefaces were taken captive by a hostile Indian tribe. According to tribe's custom they had to pass an intelligence test, or die. The chieftain showed 5 headbands – 2 red and 3 white. The 3 men were blindfolded and positioned one after another, face to back. The chief put a headband on each of their heads, hid two remaining headbands, and removed their blindfolds. So the third man could see the headbands on the two men in front of him, the second man could see the headband on the first, and the first could not see any headbands at all. According to the rules any one of the three men could speak first and try to guess his headband color. And if he guessed correctly – they passed the test and could go free, if not – they failed. It so happened that all 3 Palefaces were prominent logicians from a nearby academy. So after a few moments of silence, the first man in the line said: "My headband is ...". What color was his head band? Why?

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Solution #1:

An important thing to note is that the worker may use what you paid him previously to make change.

Divide the bar into 3 pieces using 2 cuts.

X: $\frac{1}{7}$ th of the bar

Y: $\frac{2}{7}$ th of the bar

Z: $\frac{4}{7}$ th of the bar

Day 1: Give the worker X

Day 2: Give him Y, bring back X

Day 3: Give him X

Day 4: Give him Z, bring back X & Y

Day 5: Give him X

Day 6: Give him Y, bring back X

Day 7: Give him X

Solution #2:

Let the balls be named ABCDEF. We weigh 1 ball each at a time.

If $A > B$, compare C and D.

If $C = D$, weigh A and C. If $A = C$ then ACD are heavy

$A > C$ then AEF are heavy

$A < C$ this case is not possible

If $C > D$, weigh E and F. If $E = F$ this case is not possible

$E > F$ then ACE are heavy

$E < F$ then ACF are heavy

If $C < D$ similar to above (interchange C&D)

If $A < B$ similar to above, interchange A and B

If $A = B$, compare A and C.

If $A = C$, weigh C and D. If $C = D$ this case is not possible

$C > D$ then ABC are heavy

$C < D$ then DEF are heavy

If $A > C$, weigh D and E. If $D = E$ then ABF are heavy

$D > E$ then ABD are heavy

$D < E$ then ABE are heavy

If $A < C$, weigh D and E. If $D = E$ then CED are heavy

$D > E$ then CDF are heavy
 $D < E$ then CEF are heavy

This can also be solved by just comparing (AB vs DE), (BC vs EF) and (AC vs DF). You can work out the conclusions on your own.

Solution #3:

The third paleface could see the colour of the head bands of both in front of him. He is silent which implies that atleast one of the head bands he sees on the other two is white. Second and First paleface being smart would have realised it.

In addition, if Second paleface would have seen that the First one has a Red head band he would have declared that he was wearing a White Red band.

But he remains silent, so the First Paleface concludes that he is wearing a White band.