

ABACUS – Summers Prep 2012 – Set#5

Question #1:

Once upon a time there was a thief. He was caught while trying to steal from the King's treasury. The King who was known for his eccentric verdicts gave a similar verdict in this case.

It was decided that the thief will be facing gun shots. Two bullets would be placed in revolver (has 6 slots in the bullet chamber) in successive order. The chamber will be rotated properly before taking the first shot at the thief. If the thief is still alive, he has the option to choose to spin the chamber again before the second shot or directly facing another shot without spinning.

If the first shot drew a blank, what should the thief choose – to spin the chamber or not to spin before facing the second shot?

Question #2:

Three ants are sitting at the three corners of an equilateral triangle. Each ant starts randomly picks a direction and starts to move along the edge of the triangle. What is the probability that none of the ants collide?

Question #3:

Two players A & B play a marble game. Each player has both a red and a blue marble. They present one marble to each other. If both present red, A wins Rs 3. If both present blue, A wins Rs 3. If the colours do not match, B wins Rs 2. Is it better to be A or B?

Question #4:

There are 25 horses and 5 race tracks. The three fastest horses have to be identified in the least no of races. In one race only 5 horses can participate. Find the least number of races required.

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Solution #1:

Thief should select the option to pull the trigger again without spinning.

We know that the first chamber was one of the four empty chambers. Since the bullets were placed in consecutive order, one of the empty chambers is followed by a bullet, and the other three empty chambers are followed by another empty chamber. So if the trigger was pulled again, the probability that a bullet will be fired is 1/4. If chamber was spun again, the probability that the thief would be shot is 2/6, or 1/3, since there are two possible bullets that would be in firing position out of the six possible chambers that would be in position.

Solution #2:

$P(\text{No collision}) = P(\text{All ants go in a clockwise direction}) + P(\text{All ants go in an anti-clockwise direction}) = 0.5 * 0.5 * 0.5 + 0.5 * 0.5 * 0.5 = 0.25$

Solution #3:

Assuming that they have equal probabilities of presenting Red or Blue, A has an expected payoff of Rs 1.5 and B has an expected payoff of Rs 1. So it is better to be A.

But hold on!! If it would have been like this - "If both present red, A wins Rs.3. If both present blue, A wins Rs 3. If the colours do not match, B wins Rs 2."

Then the expected payoffs for both A & B would have been Rs 1. You can stop reading right here and try to solve by yourself. Read on for the answer.

Player B has a variance of Rs 1 and A has a variance of Rs 1.5. So if you are risk averse, it is better to be Player B since it offers the same expected return, but less risk.

Solution #4:

Divide the 25 horses into 5 equal groups. Let's call these the primary groups.

Conduct 1 race for each of the primary groups to get the ranks of horses in their respective primary groups.

Conduct 1 race among the horses having Rank 1 in their primary groups. Let's call the fastest horse in this race as I, the second fastest as II and the third fastest as III.

Conduct 1 race between II, III, Ranks 2 & 3 from primary group having I, and Rank 2 from primary group having II. So I and the top 2 in the 7th race are the 3 fastest horses.

So minimum races required is 7.

The trick is to write all the ranks in a 5 x 5 tabular format after the 6th race is over and try to eliminate all the horses that cannot be the top 3. You would find that the remaining horses would be 6 including I who is the fastest one, and hence the 7th race may be conducted among the remaining 5.