abacus
THE QUANT CLUB OF IIMA

ABACUS – Summers Prep 2012 – Set#6

Question #1:

Q (a) Use four 4s to make 660.

(b) Use four 4s to make 649.

You may make use of any commonly used mathematical operators/functions

(+,-,/,^,∑, mod(), log, sin, √,!, decimal , ……..etc.)

Question #2:

Two players alternatively erase one number from the sequence 1, 2, ... , 27 until only two numbers remain. The first player wins if the sum of these numbers is divisible by 5; otherwise the second player wins. Who has a winning strategy?

Question #3:

Prove that if p is a prime number greater than 3 then, p^2 - 1 is divisible by 24

Question #4:

The restrooms on the 7th floor of Wean hall have just been renovated. Unfortunately, the contractors omitted to label the doors with Men/Women. A visitor to the Mathematical Sciences Department arrives outside the rest-rooms and does not wish to go in the wrong door. Standing outside the door are the famous Crane triplets: Ampule, Botule and Corpule. These guys are identical. Their own mother cannot tell them apart. It is well-known in the academic world that Ampule is a good person and always tells the truth, Botule is a mean person and always lies. Corpule is confused and half the time he tells the truth and the other half of the time, he lies. Our visitor knows that he is allowed two questions. What should they be? (Note that a question will be directed to one triplet who will answer it in his own way.)

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Solution #1:

660 = (4 + √4)! – 4! /.4

649 = (√4/.4) ^4 + 4!

Solution #2:

The first player A has a winning strategy. The state of the game at any time can be described by a sequence x−2, x−1, x0.x1, x2 where xj is the number of integers remaining that are equal to j mod 5. If A can arrange things so that the other player B faces a position where xj = x−j for j = 1, 2 and x0 is even then player A will win. Call such a position a balanced position. If B takes a number which is k mod 5 then in the next round A will take a number which is −k mod 5 and once again there will be a balanced position. At the end, when there are 2 numbers left and they form a balanced position, A will have won.

A begins by taking number 1. After this we almost have a balanced position, except that there is an extra 0 and 2. A plays as if in a balanced position up until B first takes a 0 or 2, in which case A will respond by taking the other choice. After this it will be a balanced position.

Solution #3:

The solution relies on showing that p^2 - 1 is a multiple of 2x2x2x3
First expand p^2 - 1 to give:
p^2 - 1 = (p - 1) x (p + 1)
Then consider the terms on the right hand side, firstly since we know that p must be odd p -1 and p + 1 must be even. so we have two of the factors we require. Additionally since p - 1 and p + 1 effectively form 2 consecutive even numbers one of them must be a multiple of 4 thus we have another of our factors of 2. So far we have 2x2x2, now to get the factor of 3
p - 1, p & p + 1 form three consecutive numbers. in any three consecutive numbers one will be a multiple of 3, we know it is not p which is a multiple of 3, as this is prime, hence either p - 1 or p + 1 is a multiple

Solution #4:

Definition: "Consistent" means someone who always tells the truth OR always tells a lie.

Observation: When you ask a consistent person "If I asked you binary question X, what would you answer?", their answer to this question will always be a correct answer to question X, thus turning a liar into a truth teller.

So, here are the two questions:

Question 1, to one of the triplets (and pointing to another one): "If I asked you if this brother is Consistent, what would you answer?". If the answer is YES, ask Question 2 of the person previously pointed to. Otherwise, ask Question 2 of the third brother. Question 2: "If I asked you if this (pointing to one of the bathrooms) is the Men's bathroom, what would you answer?". If the answer is YES, the bathroom pointed to is the men's bathroom, otherwise, it's otherwise.

Explanation: if the first brother being asked is consistent, then the second brother being asked is also consistent. If the rst brother being asked is not consistent, then both his brothers are consistent, and therefore regardless of how he answers, question 2 is asked of a consistent person.