ABACUS Tesseract 2013 – Set #7

Question #1:

A gambler goes to bet. The dealer has 3 dice, which are fair, meaning that the chance that each face shows up is exactly 1/6.

The dealer says: "You can choose your bet on a number, any number from 1 to 6. Then I'll roll the 3 dice. If none show the number you bet, you'll lose \$1. If one shows the number you bet, you'll win \$1. If two or three dice show the number you bet, you'll win \$3 or \$5, respectively."

Is it a fair game?

Question #2:

In front of you are three poles. One pole is stacked with 8 rings ranging in weight from one ounce (at the top) to 8 ounces (at the bottom). Your task is to move all the rings to one of the other two poles so that they end up in the same order. The rules are that you can move only one ring at a time, you can move a ring only from one pole to another, and you cannot even temporarily place a heavier ring on top of a lighter ring.

What is the minimum number of moves you need to achieve the task?

Question #3:

Two players take turns choosing one number at a time (without replacement) from the set {-4, -3, -2, -1, 0, 1, 2, 3, 4}. The first player, to obtain three numbers (out of three, four, or five) which sum to 0, wins.

Does either player have a forced win?

SOLUTIONS

Solution 1:

It's a fair game. If there are 6 gamblers, each of whom bet on a different number, the dealer will neither win nor lose on each deal.
If he rolls 3 different numbers, e.g. 1, 2, 3, the three gamblers who bet 1, 2, 3 each wins $1 while the three gamblers who bet 4, 5, 6 each loses $1.
If two of the dice he rolls show the same number, e.g. 1, 1, 2, the gambler who bet 1 wins $3, the gambler who bet 2 wins $1, and the other 4 gamblers each loses $1.
If all 3 dice show the same number, e.g. 1, 1, 1, the gambler who bet 1 wins $5, and the other 5 gamblers each loses $1.
In each case, the dealer neither wins nor loses. Hence it's a fair game.

Solution 2:

Let the posts be named A, B ,C. A initially has the 8 rings, C is the desired destination for moving the rings.
For moving n disks from post A to post C:
1. First, transfer n-1 disks from post A to post B. The number of moves will be the same as those needed to transfer n-1 disks from post A to post C. Call this number M[n-1] moves.
2. Next, transfer the remaining 1 disk from post A to post C [1 move].
3. Finally, transfer the remaining n-1 disks from post B to post C. [Again, the number of moves will be the same as those needed to transfer n-1 disks from post A to post C, or M[n-1] moves.]
Therefore the number of moves needed to transfer n disks from post A to post C is 2M[n-1]+1,
M[n] = 2M[n-1] + 1
On solving this recursion with base cases (M[1] = 1; M [2] = 3) we get M[n] = 2^n – 1
M[8] = 255.

Solution 3:

Consider a 3 x 3 magic square, wherein all of the rows, columns, and diagonals sum to 0; example below. It's not difficult to see that the aim of the game, as stated, can be satisfied if, and only if, the three integers fall in the same row, column, or diagonal.

1	2	-3
-4	0	4
3	-2	-1

Hence the game is equivalent to tic-tac-toe, or noughts and crosses, a game which, with best play, is well known to be a draw. Therefore neither player has a forced win.