# ABACUS Tesseract 2013 – Set #8

## Question #1:

There is an island with 100 women. 50 of the women have red dots on their foreheads, and the other 50 women have blue dots on their foreheads.

If a woman ever learns the color of the dot on her forehead, she must permanently leave the island in the middle of that night.

One day, an oracle appears and says "at least one woman has a blue dot on her forehead." All the woman know that the oracle speaks the truth. All of them are perfect logicians (and know that the others are perfect logicians too). What happens next?

## Question #2:

Once upon a time there was a thief. He was caught while trying to steal from the King's treasury. The King who was known for his eccentric verdicts gave a similar verdict in this case.

It was decided that the thief will be facing gun shots. Two bullets would be placed in revolver (has 6 slots in the bullet chamber) in successive order. The chamber will be rotated properly before taking the first shot at the thief. If the thief is still alive, he has the option to choose to spin the chamber again before the second shot or directly facing another shot without spinning.

If the first shot drew a blank, what should the thief choose – to spin the chamber or not to spin before facing the second shot?

## Question #3:

Three ants are sitting at the three corners of an equilateral triangle. Each ant starts randomly picks a direction and starts to move along the edge of the triangle. What is the probability that none of the ants collides?

**SOLUTIONS**

**Solution 1:**

We will use induction to solve this riddle.

**Base case**:

Imagine the same situation, except with 1 blue dot and 99 red dots. The woman with the blue dot will see 99 red dots, and will realize the one blue dot must be her own. She will leave on the first night.

The next day, the other 99 women will see that the one woman they all knew had a blue dot is gone. This indicates to them that their own dot cannot be blue (since the woman wouldn't have left on the first night if she'd seen any blue dots). So they all know their dots are red. All 99 of them will leave on the second night.

**Inductive step:**

Inductive hypothesis: If there are exactly N blue dots, every woman with a blue dot will leave on night N.

Now assume there are N+1 blue dots. Each woman with a blue dot will see N other blue dots, and will know that there are either N blue dots (meaning their own dot isn't blue), or that there are N+1 blue dots (meaning their own dot is blue).

After N nights, nobody will have yet left the island. By the inductive hypothesis, there could not be exactly N blue dots or else all of those women would have left on the Nth night. So each woman with a blue dot will now know that there must be N+1 blue dots, and that the last dot is their own. So every woman with a blue dot will leave on night N+1.

Then, all the women with the red dots will realize that there must have been exactly N+1 blue dots, since by the inductive hypothesis this is why all the women with blue dots left on the (N+1)th night. So these women now know that their own dots couldn't have been blue, and therefore must be red. They now know their dot colors, and will leave on the (N+2)nd night.

So for the situation described in the riddle, the 50 women with blue dots will leave on night 50, and the 50 women with red dots will leave on night 51.

**Solution 2:**

Thief should select the option to pull the trigger again without spinning.

We know that the first chamber was one of the four empty chambers. Since the bullets were placed in consecutive order, one of the empty chambers is followed by a bullet, and the other three empty chambers are followed by another empty chamber. So if the trigger was pulled again, the probability that a bullet will be fired is 1/4. If chamber was spun again, the probability that the thief would be shot is 2/6, or 1/3, since there are two possible bullets that would be in firing position out of the six possible chambers that would be in position

**Solution 3:**

P(No collision) = P(All ants go in a clockwise direction) + P( All ants go in an anti-clockwise direction) = 0.5 * 0.5 * 0.5 + 0.5 * 0.5 * 0.5 = 0.25